Now, in writing this paper, I took it as a "given" that the readership would have a general understanding of solving the Poisson equation using the 3D Green's function, but the comments from those on Prof. Tao's global regularity blog clearly indicate this is not the case. Therefore, I will provide additional details here on how we arrive at equation (116) of the article. First, note that the integral in equation (14) is an improper integral since the region of integration $\mathbb{R}^3$ includes $\mathbf{x}' = \mathbf{x}$ where the function being integrated becomes undefined. The derivation of this Green's function, which is merely standard PDE theory, indicates how this integral is to be approached. What must be done is to first define a small sphere region $\sigma(\epsilon, \mathbf{x}, t)$ of radius ϵ centered at point $\mathbf{x}$ at time t. We then perform the integral over the region $\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)$ which is merely $\mathbb{R}^3$ with the spherical region $\sigma(\epsilon, \mathbf{x}, t)$ removed. Note that the point $\mathbf{x}' = \mathbf{x}$ is excluded from the region for all $\epsilon > 0$ and therefore the resulting integral is defined. If Q is a smooth function, however, this integral will approach a finite value as $\epsilon \to 0$. But as $\epsilon \to 0$ the integration region $\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)$ reduces to $\mathbb{R}^3$. Therefore equation (14) can be written more precisely as

$$p(\mathbf{x}, t) \;=\; \frac{1}{4\pi} \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \frac{Q(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|} \, d^3\mathbf{x}' \tag{4-1}$$

Taking the gradient of this equation, we have

$$\nabla p(\mathbf{x}, t) \;=\; -\frac{1}{4\pi} \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} Q(\mathbf{x}', t) \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \, d^3\mathbf{x}' \tag{4-2}$$

We now take the absolute value of both sides and use the triangle inequality to obtain

$$|\nabla p(\mathbf{x}, t)| \;\le\; \frac{1}{4\pi} \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \frac{|Q(\mathbf{x}', t)|}{|\mathbf{x} - \mathbf{x}'|^2} \, d^3\mathbf{x}' \tag{4-3}$$

which is a more precise form of inequality (112). Henceforth in this work, the integral of a quantity over $\mathbb{R}^3$ is defined as the limit as $\epsilon \to 0$ of the integral over $\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)$ as shown above. Integrating both sides of this inequality with respect to t and changing the order of integration on the right-hand side, we then have

$$\Lambda(\mathbf{x}, t) \;=\; \int_0^t |\nabla p(\mathbf{x}, t')| \, dt' \;\le\; \frac{1}{4\pi} \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \frac{q(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|^2} \, d^3\mathbf{x}' \tag{4-4}$$

Therefore, equation (4-4) implies that no singularity in q at $\mathbf{x}' = \mathbf{x}$ will contribute to the integrals in equations (116) and (118), and hence they remain finite.

At this point, let us take a look at the rules for solving the Poisson equation according to Anonymous, Antoine Deleforge, and Gandhi Viswanathan, and see if we can put their words into an equation. According to their posts, we must first take the limit as $\epsilon \to 0$ before any integration is done. In this case, equation (4-2) becomes

$$\nabla p(\mathbf{x}, t) \;=\; -\frac{1}{4\pi} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \lim_{\epsilon \to 0} Q(\mathbf{x}', t) \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \, d^3\mathbf{x}' \tag{4-5}$$

Now, does this make any sense at all? It seems that this involves taking the limit as $\epsilon \to 0$ of the function $Q(\mathbf{x}', t)$ which doesn't even depend on ϵ. In this case, of course, the limit taking operation has no effect, and we can write equation (4-5) as

$$\nabla p(\mathbf{x}, t) \;=\; -\frac{1}{4\pi} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} Q(\mathbf{x}', t) \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \, d^3\mathbf{x}' \tag{4-6}$$

But we still have the ϵ parameter in the subscript of the integral sign for defining the region of integration. Well, we probably should set it equal to zero since Gandhi Wiswanathan claimed we would be "cheating" to ignore what happens just outside or around the point of the singularity. In this case, the region of integration simply reduces to $\mathbb{R}^3$ and this equation can be written as

$$\nabla p(\mathbf{x},t) \;=\; -\frac{1}{4\pi}\int_{\mathbb{R}^3} Q(\mathbf{x}',t)\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\; d^3\mathbf{x}' \tag{4-7}$$

At this point, we merely follow the steps of the proof shown in equations (110)-(118) of the paper, but this time with the understanding that if Q blows up at the field point $\mathbf{x}$, so does $\nabla p(\mathbf{x},t)$ and $\Lambda(\mathbf{x},t)$. This, of course, allows the integral in equation (127) to become infinite at some finite time T_b which establishes their alleged "global error" in my proof, thereby foiling the claim of a smooth solution for all $t \geq 0$.

And so, the Navier-Stokes Millennium Problem is saved from a global regularity result by creative re-solution of the Poisson equation by Anonymous, Antoine Deleforge, and Gandhi Viswanathan. In fact, we should give equation (4-5) a name! Shall we call it the *Anonymous-Deleforge-Viswanathan Theorem*, or simply the *ADV Theorem*? As we will see in the next chapter, this "theorem" implies something even more interesting when applied to the field of electrostatics. So, stay tuned!

Chapter 5

The Effect of the ADV Theorem on Electrostatics

The ADV theorem has some interesting implications in the field of Electrostatics. Before getting into specifics, however, we should probably take a refresher lesson on a few of the basics. First, a particle carrying a charge c_i sets up an electric field $\mathbf{E}_i$ (a vector quantity) which will exert a force on a neighboring charged particle if present. The electric field from this point charge is given by

$$\mathbf{E}_i(\mathbf{x}) \;=\; c_i \,\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} \qquad \text{if } \mathbf{x} \neq \mathbf{x}_i$$

where $\mathbf{x}$ is the position vector of the field point and $\mathbf{x}_i$ is the position of the charge. Now, by definition, this electric field $\mathbf{E}_i$ is the electrostatic force per unit charge acting on a test charge placed at position $\mathbf{x}$. Note that this value becomes arbitrarily large for $\mathbf{x}$ sufficiently close to $\mathbf{x}_i$. If we place that test charge exactly at $\mathbf{x}_i$, however, then this force per unit charge $\mathbf{E}_i$ becomes zero and not infinite. From the radial nature of the electrostatic force due to this charge at $\mathbf{x}_i$, we can see at least intuitively that no net force can be exerted on the test charge at this point even though we may think of the "magnitude" of $\mathbf{E}_i$ as being infinite in all directions. Therefore, we have $\mathbf{E}_i(\mathbf{x}_i) = 0$ and the field $\mathbf{E}_i(\mathbf{x})$ at all field points $\mathbf{x}$ can be summarized as

$$\mathbf{E}_i(\mathbf{x}) \;=\; c_i \,\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} \quad \text{if } \mathbf{x} \neq \mathbf{x}_i \qquad \text{and} \qquad \mathbf{E}_i(\mathbf{x}_i) = 0 \tag{5-1}$$

If we have a collection of N charges, we can obtain the total field $\mathbf{E}(\mathbf{x})$ merely by performing a vector sum over the $\mathbf{E}_i(\mathbf{x})$. That is

$$\mathbf{E}(\mathbf{x}) \;=\; \sum_{i=1}^{N} \mathbf{E}_i(\mathbf{x}) \tag{5-2}$$

and if $\mathbf{x}$ is not equal to any of the $\mathbf{x}_i$, this equation can be written as

$$\mathbf{E}(\mathbf{x}) \;=\; \sum_{i=1}^{N} c_i \,\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} \tag{5-3}$$

Now let us suppose we wish to determine the electric field at each of the point charges in our collection. From equations (5-1) and (5-2), we have

$$\mathbf{E}(\mathbf{x}_i) \;=\; \sum_{j=1}^{N} \mathbf{E}_j(\mathbf{x}_i) \;=\; \sum_{j\neq i} c_j \,\frac{\mathbf{x}_i-\mathbf{x}_j}{|\mathbf{x}_i-\mathbf{x}_j|^3} \tag{5-4}$$

The constraint $j \neq i$ is important physically as well as mathematically. It's an example of a general principle that a particle cannot exert a force on itself. Also in this equation, we see that the force on particle i due to the field set up by particle j is given by

$$\mathbf{F}_{ij} \;=\; c_i\, c_j \,\frac{\mathbf{x}_i-\mathbf{x}_j}{|\mathbf{x}_i-\mathbf{x}_j|^3} \tag{5-5}$$

which is equal and opposite to the force on particle j due to the field set up by particle i given by

$$\mathbf{F}_{ji} \;=\; c_j\, c_i\, \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^3} \;=\; -\mathbf{F}_{ij} \tag{5-6}$$

thereby verifying Newton's Third Law.

Let us now consider the $\mathbf{E}$ field from a continuum of charge instead of individual point charges. In this case, we replace the summation over point charges in equation (5-3) with an integral over a continuous distribution of charge characterized by a smoothly varying charge density $\rho_c(\mathbf{x})$. Equation (5-3) can then be written as

$$\mathbf{E}(\mathbf{x}) = \int_{\mathbb{R}^3} \rho_c(\mathbf{x}')\, \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3\mathbf{x}' \tag{5-7}$$

In writing this equation, however, we are assuming there is no charge at the field point $\mathbf{x}$. That is, $\rho_c(\mathbf{x}) = 0$. To allow for the possibility of $\rho_c(\mathbf{x}) \neq 0$, we must exclude this point from the integral, similar in principle to what we did with discrete point charges in equation (5-4). To do this, we first define the spherical region $\sigma(\epsilon, \mathbf{x})$ of radius ϵ centered at the field point $\mathbf{x}$ as we did in Chapter 4, and then perform the integral indicated in equation (5-7) over the region $\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x})$ which is simply $\mathbb{R}^3$ with the spherical σ region removed. Finally, we take the limit of this integral as $\epsilon \to 0$ to obtain

$$\mathbf{E}(\mathbf{x}) = \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \rho_c(\mathbf{x}')\, \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}\; d^3\mathbf{x}' \tag{5-8}$$

In this integral, all contributions to $\mathbf{E}$ are included from charges arbitrarily close but not equal to $\mathbf{x}$.

At this point, we define the electrostatic potential $U(\mathbf{x})$ as

$$U(\mathbf{x}) = \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \frac{\rho_c(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\; d^3\mathbf{x}' \tag{5-9}$$

Taking the gradient of both sides of this equation, we then have

$$\mathbf{E}(\mathbf{x}) = -\nabla U(\mathbf{x}) = \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - \sigma(\epsilon, \mathbf{x}, t)} \rho_c(\mathbf{x}')\, \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}\; d^3\mathbf{x}' \tag{5-10}$$

Now, if $\rho_c(\mathbf{x}')$ is bounded and smooth, then contributions to this integral from points $\mathbf{x}'$ arbitrarily close to the field point $\mathbf{x}$ are finite. That is, these points will not cause a blowup of the integral due to their close proximity to $\mathbf{x}$ which would tend to make the denominator in equation (5-10) very small. To show this, we first define the following vector field based on the integral in equation (5-10)

$$\mathbf{E}_{R\epsilon}(R, \mathbf{x}, \epsilon) = \int_{\sigma(R, \mathbf{x}) - \sigma(\epsilon, \mathbf{x})} \rho_c(\mathbf{x}')\, \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}\; d^3\mathbf{x}' \tag{5-11}$$

where $\sigma(R, \mathbf{x})$ is defined as the spherical region with radius R centered at $\mathbf{x}$. Therefore, the region of integration $\sigma(R, \mathbf{x}) - \sigma(\epsilon, \mathbf{x})$ is defined as the sperical region $\sigma(R, \mathbf{x})$ with the smaller concentric sperical region $\sigma(R, \mathbf{x})$ removed. Also, this same region could be specified as the region between two concentric sperical shells (centered at point $\mathbf{x}$), with radii ϵ and R. Taking the absolute value of both sides of this equation and using the triangle inequality, we then have

$$|\mathbf{E}_{R\epsilon}(R, \mathbf{x}, \epsilon)| \leq \int_{\sigma(R,\mathbf{x})-\sigma(\epsilon,\mathbf{x})} |\rho_c(\mathbf{x}')| \frac{|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|^3} \, d^3\mathbf{x}' = \int_{\sigma(R,\mathbf{x})-\sigma(\epsilon,\mathbf{x})} \frac{|\rho_c(\mathbf{x}')|}{|\mathbf{x}-\mathbf{x}'|^2} \, d^3\mathbf{x}' \tag{5-12}$$

If we now define the set of primed spherical coordinates r', θ', and ϕ' such that

$$\mathbf{x}' = \mathbf{x} + r' \sin\theta' \cos\phi' \mathbf{e}_1' + r' \sin\theta' \sin\phi' \mathbf{e}_2' + r' \cos\theta' \mathbf{e}_3' \tag{5-13}$$

where $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are the unit basis vectors for the coordinate system, this equation can be written as

$$|\mathbf{E}_{R\epsilon}(R, \mathbf{x}, \epsilon)| \leq \int_{\epsilon}^{R} \int_0^{2\pi} \int_0^{\pi} \frac{1}{r'^2} |\rho_c(\mathbf{x}'(r', \phi', \theta'))| \, r'^2 \sin\theta' \; d\phi' \, d\theta' \, dr' \tag{5-14}$$

$$= \int_{\epsilon}^{R} \int_0^{2\pi} \int_0^{\pi} |\rho_c(\mathbf{x}'(r', \phi', \theta'))| \, \sin\theta' \; d\phi' \, d\theta' \, dr' \qquad \text{if } \epsilon > 0$$

Taking the limit as $\epsilon \to 0$ of this inequality, we then have

$$\lim_{\epsilon\to 0} |\mathbf{E}_{R\epsilon}(R, \mathbf{x}, \epsilon)| \leq \int_0^{R} \int_0^{2\pi} \int_0^{\pi} |\rho_c(\mathbf{x}'(r', \phi', \theta'))| \, \sin\theta' \; d\phi' \, d\theta' \, dr' \tag{5-14}$$

and since ρ_c is a bounded function, its integral over a finite range of coordinates must be finite. Therefore, if we have a bounded charge density function $\rho(\mathbf{x}')$, then the integral in equation (5-10) cannot blow up due to charged volume elements in close proximity to the field point.

We now consider the problem of determining the **E**-field due to a point-charge singularity in ρ_c at the field point $\mathbf{x}$. The electrostatic potential $U(\mathbf{x})$ is given by

$$U(\mathbf{x}) = \frac{c}{|\mathbf{x}-\mathbf{x}'|} = \frac{c}{r'} \tag{5-15}$$

In this case, the boundary of the spherical region $\sigma(\epsilon, \mathbf{x})$ is an equa-potential surface (in the $\mathbf{x}'$ coordinates) of value $U = c/\epsilon$. This means that

$$\int_{\sigma(\epsilon,\mathbf{x})} \nabla' U(\mathbf{x}') \, d^3x' \; = \; -\int_{\sigma(\epsilon,\mathbf{x})} \mathbf{E}(\mathbf{x}') \, d^3x' = 0 \tag{5-16}$$

for all $\epsilon > 0$, and therefore the mean value of $\mathbf{E}$ over $\sigma(\epsilon, \mathbf{x})$ is given by

$$\langle \mathbf{E}(\mathbf{x}') \rangle_{\sigma(\epsilon,\mathbf{x})} = \frac{3}{4\pi\epsilon^3} \int_{\sigma(\epsilon,\mathbf{x})} \mathbf{E}(\mathbf{x}') \, d^3x' = 0 \tag{5-17}$$

This mean value $\mathbf{E}$ must approach the value of $\mathbf{E}$ at the center $\mathbf{x}$ of the region $\sigma(\epsilon, \mathbf{x})$ as $\epsilon \to 0$. Therefore, we have

$$\mathbf{E}(\mathbf{x}) \; = \; \lim_{\epsilon\to 0} \langle \mathbf{E}(\mathbf{x}') \rangle_{\sigma(\epsilon,\mathbf{x})} \; = \; 0 \tag{5-18}$$

Thus, we see that the value of $\mathbf{E}$ at the field point $\mathbf{x}$ is not affected by a point charge placed at that point. Therefore, the value of $\mathbf{E}$ at the field point $\mathbf{x}$ is given by equation (5-10), regardless of whether a finite point charge is placed at this point.

Now, these principles have been applied successfully to electrostatics problems for well over a century, but the more recent ADV Theorem, which seems to have been developed primarily to refute proofs of global regularity of the Navier-Stokes equation, gives a somewhat different concept of the self-field of a charged particle. In going from establishing the scalar pressure p and its gradient in the Navier-Stokes

Millennium Problem to the elementary electrostatics results shown above, however, we merely make the following substitutions

$$\text{Scalar Pressure } p \to \text{Electrostatic Potential } U$$
$$\text{Pressure Gradient } \nabla p \to \nabla U = \text{Negative of Electric Field } -\mathbf{E}$$
$$Q \to \text{Electrostatic Charge Density } \rho_c$$

and the mathematics is otherwise identical. Therefore, we insert the charge density $\rho_c(\mathbf{x}') = c\,\delta(\mathbf{x}-\mathbf{x}')$ corresponding to a point charge into the ADV Theorem to obtain

$$\mathbf{E}(\mathbf{x}) = \int_{\mathbb{R}^3} c\,\delta(\mathbf{x}-\mathbf{x}')\,\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\;d^3\mathbf{x}' = c\int_{\mathbb{R}^3} \frac{\delta(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\,\mathbf{e}_r\;d^3\mathbf{x}' \tag{5-19}$$

where δ denotes the Dirac δ function and $\mathbf{e}_r$ is a unit vector in the direction of $\mathbf{x}-\mathbf{x}'$. At this point, it is quite evident that we are arriving at some "vector" quantity that is infinite in magnitude and totally undefined in direction. The point $\mathbf{x}' = \mathbf{x}$ was not excluded from the region of integration since the ADV Theorem takes all limits "in the correct order" and therefore set $\epsilon \to 0$ before introducing any δ functions describing the charge density ρ_c. Furthermore, Gandhi Viswanathan himself stated that it would be "cheating" to ignore this singularity even though it was shown in equation (5-18) that the electric field at point $\mathbf{x}$ due to a point charge at this same point $\mathbf{x}$ vanishes.

So, for the first time ever, it has been predicted that a point charge is accelerated by its own self-field. Not only that, but it is a field of infinite magnitude. This discovery could conceivably be the key to limitless supplies of energy! My congratulations to the Anonymous-Deleforge-Viswanathan team! I recall in his posting to the blog on 23 February 2022 at 8:17 AM, Dr. Viswanathan stated to me that "...You might as well also claim that you can turn water into wine!!!!!". Well, it seems as though Dr. Viswanathan has performed his miracle also, but of a somewhat different form. – LOL! Or, is this simply another aspect of the Navier-Stokes Millennium Problem research that needs some re-thinking?